Exercise 28

Differentiate f and find the domain of f.

$$f(x) = \sqrt{2 + \ln x}$$

Solution

Recognize that only the square root of a nonnegative number can be taken, and only the logarithm of a positive number can be taken.

$$2 + \ln x \geq 0 \quad \text{and} \quad x > 0$$

$$\ln x \geq -2 \quad \text{and} \quad x > 0$$

$$x \geq e^{-2} \quad \text{and} \quad x > 0$$

Therefore, the domain of the function is

$$[e^{-2}, \infty).$$

Take the derivative of the function with respect to x by using the chain rule.

$$\begin{aligned} f'(x) &= \frac{d}{dx}\left(\sqrt{2 + \ln x}\right) \\ &= \frac{1}{2}(2 + \ln x)^{-1/2} \cdot \frac{d}{dx}(2 + \ln x) \\ &= \frac{1}{2}(2 + \ln x)^{-1/2} \cdot \left(\frac{1}{x}\right) \\ &= \frac{1}{2x\sqrt{2 + \ln x}} \end{aligned}$$